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EXCHANGE RATE, INCOME DISTRIBUTION AND TECHNICAL CHANGE IN A BALANCE-OF-PAYMENTS CONSTRAINED GROWTH MODEL

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Resumo

Este trabalho desenvolve um modelo formal capaz de avaliar o efeito líquido do impacto de uma desvalorização cambial sobre a razão entre as elasticidades-renda da demanda por exportações e importações e, consequentemente, sobre o crescimento restringido pelo balanço de pagamentos no longo prazo. Tal modelo mostra como a taxa de câmbio impacta sobre o padrão d especialização do país via mudanças nas variáveis da estrutura econômica, como a distribuição funcional da renda e o nível tecnológico do país. O modelo se baseia em duas hipóteses principais. Primeiramente, o modelo assume que inovações tecnológicas impactam positivamente na elasticidade-renda das exportações e negativamente na elasticidade-renda das importações. A outra hipótese afirma que um aumento na participação dos salários na renda reduz a elasticidade-renda das importações. Por fim, o modelo mostra como o impacto do câmbio sobre o crescimento de longo prazo é ambíguo e depende de uma série de condições.

Palavras-chave: distribuição de renda, progresso tecnológico, taxa real de câmbio e crescimento restringido pelo balanço de pagamentos.

Abstract

This paper sets out a formal model to account for what would be the net effect of an exchange rate undervaluation on both income elasticities of demand for export and imports and, consequently, on the long-term balance-of-payments constrained growth rate. Such a model shows how the exchange rate impacts on the home country specialization pattern via changes in the variables of the economic structure such as the income distribution and technological change. It is built upon two basic hypotheses. Firstly, it assumes technological improvements impact positively on income elasticity of demand for exports and negatively on income elasticity of demand for imports. Secondly, it assumes here that improvements in income distribution reduce income elasticities of demand for imports. The model shows that the net impact of a currency devaluation on growth is ambiguous and depends on several conditions.

Keywords: income distribution, technological progress, real exchange rate and balance-of-payments constrained growth.

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I. INTRODUCTION

A topic of main concern among economists nowadays is the impacts of exchange rate on growth. We might say the exchange rate is, in an open economy, one of the most important macroeconomic policy tools due to its capacity of reflecting and changing, to a certain extent, the way countries relate to each other in the international trade. After the Word War II, Keynes strongly advocated the adoption of fixed exchange rates within the rules set forth in the Bretton Woods agreement. In the early 1950s the fixed exchange rate system worked pretty well, for the US had the major share of the world's official gold reserves. However, during the 1960s this scenario began to drastically change. The low interest rates employed by the US government drained out their gold reserves, thereby reducing their financial account surplus. Besides, the Vietnam War-related expenditures and US foreign aid to other countries were pointed out as the main cause of the country's ever-growing fiscal and current account deficits. Lastly, the US also had to struggle with a sharp increase in the price of some inputs and the 1973 oil crisis (Glyn *et al*, 1990).

As a result of these events, the Bretton Woods system fell apart. In this moment, the majority of the economists began to claim countries would be better off with a floating exchange rate regime instead. They believed that such a regime would correct real exchange rate (RER) misalignments, thereby preventing balance-of-payments crisis and bringing autonomy back to monetary policy. According to the orthodox monetarist theory, in an open economy with a floating exchange rate system, the balance-of-payments is, by definition, always in equilibrium. It happens because, given any foreign exchange fluctuation, the money exchange rate and the domestic prices adjust themselves automatically, thus keeping the RER constant and the balance-of-payments in equilibrium. Underlying such an argument there is a strong belief that free self-regulating markets, instead of governments for instance, can determine more efficiently the equilibrium exchange rate yielding the best possible result for the economy as a whole. Therefore, the floating exchange rate regime should be the best way to avoid balance-of-payments crisis and promote economic growth. Thus, it is clear that, according to this school of thought, the exchange rate adjusts itself endogenously and, consequently, can play no role as a macroeconomic policy tool. It means only market is in charge of correcting any possible misalignments and hence boosting growth.

Nonetheless, in the last few years a vast range of empirical evidences demonstrating a meaningful connection between exchange rate volatility and a poor growth performance for several countries since the adoption of the floating exchange rate regime in the 1970s has been brought to light (Dollar, 1992; Razin & Collins, 1997; Aguirre & Calderón, 2005). The empirical literature also shows that undervaluation/overvaluation periods are associated, by and large, to a higher/lower-growth pattern (Sachs, 1985; Dollar, 1992; Loayza *et al*, 2004; Aguirre & Calderón, 2005; Rodrik, 2007, Gala 2007). However, in spite of the fair amount of empirical analysis, there are very few attempts in the literature to provide a theoretical framework addressing the problem.

Among the mainstream economists, the main theoretical work about the relation between exchange rate and long-term growth is developed by Rodrik (2007). He argues that, in developing countries, a week tradables sector would be the cause of overvalued RER and low growth rates. The very explanation, according to the author, lies in the existence of poor institutions and market failures in these countries affecting negatively more the tradables sector than the non-tradables sector. In such a context, an undervaluation would be a second-best solution to foment growth, once a currency depreciation raises the tradables profitability.

Regarding the relation between exchange rate and growth, under a Keynesian-Kaleckian-Kaldorian perspective, we can point out some theoretical works. Blecker (1989, 1999) developed a model relating income distribution and capital accumulation for open economies in the short run. In his model, the relationship between the RER and the income distribution depends on the mark-up and on the ratio of labour cost to import prices in domestic currency. The ultimate impact of a depreciation on income distribution and capital accumulation will depend on the parameters of the model. Bhaduri & Marglin (1990) developed a short-term framework capable of accounting for a wage-led as well as a profit-led regime of accumulation in a more general Keynesian-Kaleckian theoretical scheme. Extending the model for an open economy, the authors came to the conclusion that, in a profit-led regime, if a depreciation leads to a lower real wage, then the international price competitiveness will be improved and hence the capital accumulation. In a wage-led regime, otherwise, the impact of an undervaluation is ambiguous. Bresser-Pereira & Nakano (2003) and Gala (2008) claim that an undervaluation raises the profit rate by reducing the real wage, hence accelerating the capital accumulation and, consequently, the long-term growth. We also have the works stating that an undervaluation, via Verdoorn coefficient, improves exports competitiveness, accelerates technological progress and modifies the country's specialization pattern through changes in the income elasticities of demand for exports and imports ratio, therefore boosting long-term growth (Barbosa-Filho, 2006; Gala & Libânio, 2008). Porcile & Lima (2010) proposed a model where the RER actually plays an important role in a BPC growth model. In their model the level of employment and the elasticity of labour supply can prevent the currency appreciation, thereby impacting directly on income distribution and growth in developing countries. Razmi, Rapetti and Skott (2012) presented a two-sector (tradables and non-tradables) model assuming the existence of (under and) unemployment. According to their formal framework, the effect of a currency devaluation on accumulation is ambiguous. On the one hand, a devaluation increases the profitability of tradable sector, which is assumed to be the only sector employing capital goods. On the other hand, it stimulates employment and raises the wages in the tradables sector. This means the impact of a devaluation on steady accumulation and long-term growth can go either way. Lima & Porcile (2012) developed a short-term growth model that takes into account the joint determination of income distribution and RER. In the medium run, the model analyses the RER stability resulting from an interclass and intraclass conflict over a preferred RER between the government, workers and capitalists.

The relationship between exchange rate and growth is undoubtedly important. However, the literature has been ignoring the role played by the dynamics between technological progress and income distribution in determining the specialization pattern of the economy. More importantly, the previous works have not convincingly explained yet the direct links between a devalued currency and improvements in non-price competitiveness of domestic goods expressed in terms of an increase in the ratio of the income elasticity of demand for exports to the income elasticity of demand for imports. Thus, our model contributes to the literature by exploring two transmission channels through which the RER impacts on the equilibrium: (i) technical change; and (ii) income distribution. That is to say, we set out a theoretical framework that accounts for the impact of a devaluation on long-term growth in a BPC growth model via changes in technological progress and in income distribution. Such a model intends to lay out the conditions under which a currency devaluation can be used by policymaker to promoting industrialization, and hence achieve higher equilibrium growth rates in the long run. Thirlwall (2011) states "that a once-for-all depreciation (or devaluation) will not put a country on a permanently higher growth rate. For this to happen, the depreciation would either have to be continuous, or affect the parameters of the model favourably" (Thirlwall, 2011, p. 16). Our model intends exactly to show the channels through which a once-for-all depreciation can affect the parameters of the model – the income elasticities – and hence boost or harm growth in the long run.

The next section presents the revisited version of the BPC growth model and discusses the dynamics between technical change and income distribution. Section III shows the impact of a currency devaluation on the dynamics between technological progress and income distribution. Section IV lays out the conditions under which a devaluation can boost or harm long-term growth. Finally, section V summarize and draw some conclusions.

II. THE MODEL

The balance-of-payments constrained growth model revisited

Let us assume the global economy consists of basically two different countries: a rich foreign country and a poor home country. The foreign country is a large economy that issues the international currency and the home country is a small economy facing a balance-of-payments constraint. There is no international capital flows between the two countries. The foreign country is a two-sector economy which produces and exports consumption goods and raw materials. The home country is a one-sector economy that produces and exports only one sort of consumption good and there is imperfect substitutability between the foreign and domestic consumption goods. It is also assumed that the home country imports consumption goods and intermediate inputs from the foreign country. That is to say, the home country imports are disaggregated in two different categories, namely, imported consumption good (M^c) and imported raw materials (M^r), that is, $M = M^c + M^r$. By doing so, we have now an extended balance-of-payments identity

$$P_d X = E\left(P_f M^c + P_f^r M^r\right) \tag{1}$$

where X is the quantity of exports; P_d is the domestic prices; P_f is the imported consumption goods prices in foreign currency; and P_f^r is the imported raw material prices in foreign currency; and E is the nominal exchange rate. Hereafter, let us assume $P_f = P_f^r$ over time. In rates of change

$$p_d + x = e + p_f + \theta_c m^c + (1 - \theta_c) m^r \tag{2}$$

where $\theta_c = M^c/M$; $(1 - \theta_c) = M^r/M$; and $p_f = p_f^r$.

The export demand function is

$$X = \left(\frac{P_d}{P_f E}\right)^{\eta} Z^{\varepsilon} \qquad \eta < 0, \varepsilon > 0 \tag{3}$$

where Z is the level of the foreign income; η is the price elasticity of demand for exports (here the own and cross price elasticities are assumed to be equal in absolute values); ε is the income elasticity of demand for exports. In rates of change

$$x = \eta\left(p_d - p_f - e\right) + \varepsilon z \tag{4}$$

The import demand functions are

$$M^c = M_0^c \left(\frac{E P_f}{P_d}\right)^{\psi} Y^{\pi_c} \qquad \psi < 0, \pi_c > 1 \tag{5}$$

$$M^r = \mu Y \tag{6}$$

where Y is the level of domestic income; M_0^c and μ are constants; ψ is the price elasticity of demand for imports; and π_c is the income elasticity of demand for imported consumption goods. Equation (5) is a stable multiplicative demand function for imported consumption goods, whereas equation (6) expresses a linear relationship – one can view it as a linear approximation to a long run multiplicative function at a specific point in time. It will be assumed here that technological innovations are neutral with respect to the amount of raw materials utilized in the production process, and hence the ration of imported raw materials to domestic output (M^r/Y) does not change over time. In this scenario, it is reasonable to assume the coefficient μ is constant.

In rates of change

$$m^c = \psi(e + p_f - p_d) + \pi_c y \tag{7}$$
$$m^r = y \tag{8}$$

Now let us extend the BPC growth model by making domestic product prices a function of exchange rate and foreign prices. To do so, at first the unit variable costs must be disaggregated in two parts, namely the unit labour costs and unit imported raw material costs:

$$P_d = T\left(\frac{W}{a} + P_f E\mu\right) \tag{9}$$

where $P_f E\mu$ is the unit imported intermediate input cost in domestic currency, and W/a is the unit labour cost, where W is the level of the nominal wage; and a is the labour productivity. In rates of change:

$$p_d = \tau + \varphi(w - \hat{a}) + (1 - \varphi)(p_f + e) \tag{10}$$

where $\varphi = P^W/(P^E + P^W)$.

Now we must show the process of relative prices adjustment. The exchange rate pass-through mechanism states that a nominal devaluation increases the rate of change of imported raw material prices in domestic currency, thereby feeding through into the rate of change of domestic prices with a lag. From equation (10), we can point out two other transmission channels through which a permanent positive shock in the nominal exchange rate can impact on the domestic prices: (i) the rate of change of money wage increases with a lag to match the increase in the inflation of domestic prices until the rate of change of the real wage equals the rate of change of labour productivity; (ii) a currency devaluation increases the market share for domestic goods, thus allowing exporters to raise their mark-up factor and, consequently, domestic prices.

To explain changes in the real wage growth rate, we have the following expression:

$$w - p_d = \alpha\hat{a} \tag{11}$$

where $\alpha = W/W^e$, where W^e is the level of nominal wage expected or desired by workers. The parameter α measures the degree of wage resistance. If there is total wage resistance, then $\alpha = 1$. The higher factor such as the level of employment and the degree of trade union bargaining power, the higher the parameter α will be. Since we are assuming the money wage increases with a lag after an inflationary shock in the domestic prices, we can say that in the short run $\alpha < 1$. However, as the growth of real wages match the rate of change of labour productivity, α converges asymptotically towards unity $(\alpha \to 1)$.

The second transmission channel concerns the mark-up growth rate, which can be defined as follows:

$$\tau = \omega(\varphi^e - \varphi) \tag{12}$$

where φ^e is the unit labour cost share in variable production costs expected or desired by workers; and $\omega > 0$ is the speed of adjustment of φ towards its equilibrium value φ^e. The higher the parameter ω, the higher the nominal wage flexibility. Let us now consider the effect of a devaluation on the mark-up growth rate and the level of the share φ over time. Before we start the analysis, let us rewrite the unit labour cost share as $\varphi = (W/P_d a)/\left[\left(P_f E/P_d\right)\mu + (W/P_d a)\right] = \sigma_L/(\sigma_{M^r} + \sigma_L)$, where σ_L and σ_{M^r} are the wage and the imported raw material shares in income, respectively. Workers bargain for a higher wage share in total income. However, as we have shown, the gap between the wage share expected by workers and the current wage share can be proxied by the gap between the unit labour cost share desired by workers and the actual unit labour share in unit variable costs. Continuing the analysis of equation (12), we can say that shortly after a nominal depreciation, since the money wage is not perfectly flexible, the share φ declines. The rise in the domestic prices caused by the inflationary impact of the depreciation on imported intermediate inputs in domestic currency leads the workers to collectively bargain a money wage increase. From equation (12), initially, we have the share φ below its equilibrium value and, consequently, an increase in the mark-up growth rate ($\tau > 0$). In the long run, as the money wage increases, the share φ converges towards its equilibrium value φ^e, and the mark-up factor does not change ($\tau = 0$). For a constant share φ, the equality $p_d = w - \hat{a} = e + p_f$ holds ($\bar{\varphi} \Rightarrow p_d = e + p_f$, see appendix I), which means that the relative prices do not change. It is worth noting, however, even though the growth of the real wage matches the increase in the labour productivity in the long run ($w - p_d = \hat{a}$), nothing can guarantee that φ^e will remain constant over time. If, for instance, after a devaluation the increase in labour productivity was caused by the adoption of labour-saving technology in large scale by domestic firms, then the long-term level of employment will drop, hence reducing the degree of trade union bargaining power. In this scenario, the equilibrium value φ^e will fall, and the share φ will converge to a lower level than its former level before the devaluation. That is to say, in this context, workers will not be able to maintain the initial real wage level after the devaluation and the wage share in income will stabilize at a lower level when relative prices cease to change. Alternatively, if the technology employed during this period of relative prices adjustment is labour-neutral, then the real wage will restore its former level when the relative prices are fully adjusted, and the share φ will not change in the long run ($\varphi^e = \varphi$). Lastly, it is worth noting that if the money wage is perfectly flexible (meaning $\omega \to \infty$), then the share φ do not change, and hence the possibility of a currency devaluation even in the short term is completely ruled out (see appendix I).

Finally, we have to define the growth of the labour productivity. According to the Verdoorn's Law, the growth of the labour productivity is positively related to the growth of output. Drawing upon the works of Kaldor (1966) and Dixon & Thirlwall (1975), we have the following equation:

$$\hat{a} = a_0 + \lambda y \tag{13}$$

where a_0 is rate of autonomous productivity growth; and $\lambda > 0$ is the Verdoorn coefficient.

That said, if we substitute (12) and (11) in (10) we have $p_d = \omega(\varphi^e - \varphi) + \varphi[p_d + (\alpha - 1)\hat{a}] + (1 - \varphi)\left(p_f + e\right)$. Rearranging the terms and assuming in the long term $\alpha \to 1$ and $\varphi \to \varphi^e$, we obtain $p_d = e + p_f$. This way we can conclude that relative prices do not change in the long run. Substituting then (13), (12), (11), (10), (8), (7) and (4) in (2), we have the revisited model:

$$y = \frac{(1-\varphi)\varepsilon z + (1+\eta+\theta\psi)[\omega(\varphi^e - \varphi) + \varphi(\alpha - 1)a_0]}{(1-\varphi)\pi - (\alpha-1)\lambda\varphi(1+\eta+\theta\psi)} \tag{14}$$

where $\pi = \theta(\pi_c) + (1-\theta)(1)$ and $(1-\varphi)\pi - \lambda\varphi(1+\eta+\theta\psi) > 0$ is the stability condition, which is invariably satisfied if the Marshall-Lerner condition holds. Accordingly, if $\alpha \to 1$ and $\varphi \to \varphi^e$ in the long run, then we can derive the Thirlwall's Law from the ELBPC growth model: $y = (1-\varphi)\varepsilon z/(1-\varphi)\pi = \varepsilon z/\pi$.

Now we finish the exposition of the ELBPCR growth model. As we have seen, this model allows us to reconciliate both EL and BPC growth models in a more general mathematical specification than the one set out by Thirlwall & Dixon (1979). The following section dedicates to the developments concerning the role played by the RER in the ELBPCR model.

Endogenizing the income elasticities

In this analysis we intend to consider the net effect of a currency undervaluation on the home country specialization pattern – here expressed by the income elasticities of demand for exports and imports ratio – via structural changes in the technology gap and income distribution. To commence, let us present the two basic hypothesis of the model concerning the impact of technological improvements and an increased wage share on the elasticities ratio. The hypotheses are twofold:

(i) It is assumed here that technological innovations impact positively on income elasticity of demand for exports and negatively on income elasticity of demand for imports. A devaluated exchange rate improves exports competitiveness, on the one hand, which accelerates technological progress pace and prompts the economy to specialization pattern changes. It also, on the other hand, induces to import substitution of high-tech products by increasing the non-price competitiveness of the domestic goods. An undervalued currency modifies the profit share in income which might lead to more (less) investments in R&D and boost (harm) technological progress.

(ii) We also assume here that improvements in income distribution reduce income elasticities of demand for imports. We can argue that an increase in the wage share at expense of the profit share in income would reduce capitalists' saving and consequently their capability of consuming superfluous and highly technological foreign products. This argument draws upon the work of Furtado (1968). In other words, a currency undervaluation can lead to a change in the consumption pattern via income distribution modifications.

The inverse of the technology gap between foreign and home countries (S) is given by

$$S = \frac{a}{a_f}, \quad S \in (0,1) \tag{15}$$

where a_f is the foreign labour productivity that grows at an exogenously given rate of change and a is the domestic labour productivity.

In an economy that allows for imported raw materials into the variable production costs, the wage share in income can be defined as follows

$$\sigma_L = \frac{W}{P_d}\frac{1}{a} = 1 - \sigma_K - \left(\frac{EP_f}{P_d}\frac{1}{\mu}\right) = 1 - \sigma_K - \sigma_{M^r} \tag{16}$$

where σ_K is the shares of profits in income, respectively. Once we know σ_L and σ_{M^r}, then σ_K is determined as a residue.

According to the hypothesis aforementioned, we have:

$$\varepsilon = \varepsilon(S) \qquad\qquad \pi = \pi(S, \sigma_L) \tag{17}$$

where $\varepsilon_S > 0$, $\pi_S > 0$ and $\pi_{\sigma_L} < 0$.

Assuming a linear function for expositional ease we obtain

$$\frac{\varepsilon}{\pi} = \beta_0 + \beta_1\sigma_L + \beta_2 S \tag{18}$$

Therefore, we have a positive relationship between the income elasticities ratio, on the left-hand side of the equation, and technical change and wage share, on the right-hand side. However, in order to analyse the net effect of RER on the elasticities ratio and, consequently, on long-term growth rate, we must describe how technological innovation and income distribution interact over time and also what is the very role of the RER in such a dynamics. Before we continue the analysis, it is worth noting that, according to equation (14), the income elasticity of demand for total imports (π) is given by the weighted average of income elasticities of demand for imported consumption goods (π_c) and imported raw materials (which is equal to unity). In this model it will be assumed that the impact of changes in the RER and in the growth rate on the weight θ is negligible. Such assumption, which can be relaxed at the expense of simplicity, guarantees that a currency devaluation can only impact on the elasticities ratio via changes in the technology gap and wage share in income.

Technological progress

At this point, we need to lay down a few more hypothesis. Following Porcile *et al* (2007) and Lima (2004), firstly, let us assume there are technology disparities between the foreign and the home countries. The foreign country pushes forward the technological frontier while the home country is lagging behind. Secondly, we assume the higher the technology gap the easier it is for the home country to catch up. Thirdly, there are two classes in the economy, capitalists and workers. Since we are assuming workers consumption equals to wages, the economy saving is the capitalists' saving. Fourthly, since there is no financial market in the model, investments in technology are funded by the profit rate. However, we assume a non-linear relation between profit rate and technological progress. At low levels of profit rate capitalists have incentives to invest in technology but run into saving shortage to do so. As the profit share in income increases capitalists raise their capacity to innovate and the technology gap reduces until the point where higher profit rates become ineffective to boost technological progress. At high levels of profit share in income, capitalists lack the incentives to introduce new technologies.

Based on these hypotheses, the technological progress equation can be expressed as

$$s = \alpha_0 + \alpha_1\sigma_K - \alpha_2\sigma_K^2 - \alpha_3 S \tag{19}$$

where s is the rate of change of S; and $\alpha_0 \lesseqgtr 0$ and $\alpha_1, \alpha_0, \alpha_3 > 0$ are parameters. Substituting σ_K with its very definition $(1 - \sigma_L - \sigma_{M^r})$ in (19) and assuming $\alpha_1 = \alpha_2$ for convenience, we have after some rearrangements

$$s = \rho_0 + \rho_1 \sigma_L - \alpha_2 \sigma_L^2 - \alpha_3 S \tag{20}$$

where $\rho_0 = \alpha_0 + \alpha_2\left(\sigma_{M^r} - \sigma_{M^r}^2\right)$ and $\rho_1 = \alpha_2(1 - \sigma_{M^r})$. See appendix II. We are also assuming henceforth $\rho_0 < 0$ without loss of generality, which means α_0 is negative and sufficiently large in absolute value. By considering $\alpha_0 < 0$, we are assuming the autonomous technological progress in the foreign country grows faster than in the home country.

The *locus* $s = 0$ describes the relation between technological progress and wage share, provided a constant technology gap over time. This equation is given by

$$S = \frac{\rho_0}{\alpha_3} + \frac{\rho_1}{\alpha_3}\sigma_L - \frac{\alpha_2}{\alpha_3}\sigma_L^2 \tag{21}$$

Accordingly, we can picture an inverted U-shaped curve relating wage share and technological progress. Taking the first derivative we obtain the point of maximum

$$\sigma_L^* = \frac{\rho_1}{2\alpha_2} = \frac{1 - \sigma_{M^r}}{2} \tag{22}$$

since $\sigma_{M^r} \in (0,1)$, then $\sigma_L^* \in (0,1/2)$. Once the second derivative is negative, $-2\,\alpha_2/\alpha_3 < 0$, then σ_L^* is a point of maximum. Now we have to impose some constraints over the parameters in order to obtain meaningful results within the domain of the variables under discussion, that is, $S, \sigma_L \in (0,1)$. Assuming equation (21) has two distinct real roots, we have it pictured on Figure 1 below. See appendix III.

Functional Income Distribution

The proportionate rate of change of the real wage depends positively on the rates of change of labour productivity and the gap between the wage share expected or desired by workers (σ_L^e) and the current wage share, as follows

$$w - p = \hat{a} + \theta(\sigma_L^e - \sigma_L) \tag{23}$$

where $\theta > 0$ is an adjustment parameter. In the long run, real wage and labour productivity will both grow at the same rate. Once $\hat{\sigma}_L = w - p - \hat{a}$, rearranging the terms, we obtain

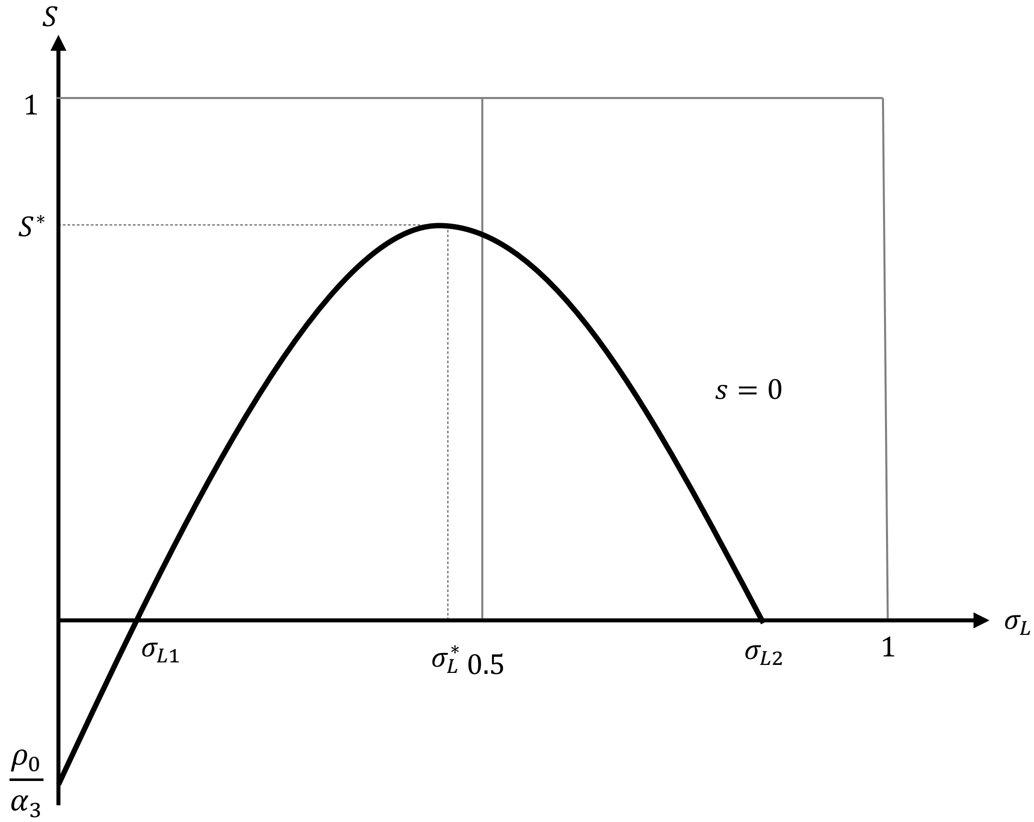
$$\hat{\sigma}_L = \theta(\sigma_L^e - \sigma_L) \tag{24}$$

Provided that, we must determine σ_L^e. It can be done from its own definition

$$\sigma_L^e = 1 - \sigma_K^e - \sigma_{M^r} \tag{25}$$

where σ_K^e is the residual expected profit share with respect to the wage share expected by workers (σ_L^e) which, by its turn, depends on the workers' bargaining power. Once σ_{M^r} is constant at first and exogenously given, that is, neither workers nor capitalists have any influence on the imported raw materials share in variable costs of production, then we can say σ_L^e and σ_K^e are inversely related.

FIGURE 1 – The *locus* $s = 0$



Therefore, now we must determine σ_K^e. Once σ_L^e depends positively on workers' bargaining power, σ_K^e, conversely, is negatively related to workers' bargaining power

$$\sigma_K^e = \omega_0 - \omega_1(l - n) \tag{26}$$

where l is the rate of change of employment, n is the exogenously given potential workers growth rate and $\omega_0, \omega_1 \geq 0$ are parameters. The underlying assumption in equation (26) is that the higher the employment rate, the higher the workers' bargaining power. Plugging equations (26) and (25) into (24) we have

$$\hat{\sigma}_L = \theta[1 - \omega_0 - \omega_1(l - n) - \sigma_{M^r} - \sigma_L] \tag{27}$$

From the definition of the long term rate of change of labour productivity, we have the following identity

$$l = y_{BP} - \hat{a} \tag{28}$$

Substituting equations (18) in the Thirlwall's Law $y = \varepsilon z/\pi$, we have

$$y_{BP} = (\beta_0 + \beta_1\sigma_L + \beta_2 S)z \tag{29}$$

Also by definition, taking the rate of change of equation (17), we obtain

$$\hat{a} = s + \hat{a}_N \tag{30}$$

Substituting (20) into (30), we have

$$\hat{a} = \rho_0 + \rho_1\sigma_L - \alpha_2\sigma_L^2 - \alpha_3 S + \hat{a}_N \tag{31}$$

Now, plugging (31) and (29) into (28) and then into (27), we obtain

$$\hat{\sigma}_L = \theta\{1 - \omega_0 + \omega_1[(\beta_0 + \beta_1\sigma_L + \beta_2 S)z - \rho_0 - \rho_1\sigma_L + \alpha_2\sigma_L^2 + \alpha_3 S + \hat{a}_N - n] - \sigma_{M^r} - \sigma_L\} \tag{32}$$

In the long term we expect the real wage and the labour productivity will grow at the same rate. Therefore, in the *locus* $\hat{\sigma}_L = 0$, the equation that describes the relationship between technology gap and wage share is given by

$$S = -\frac{\gamma_0}{\gamma_3} - \frac{\gamma_1}{\gamma_3}\sigma_L - \frac{\gamma_2}{\gamma_3}\sigma_L^2 \tag{33}$$

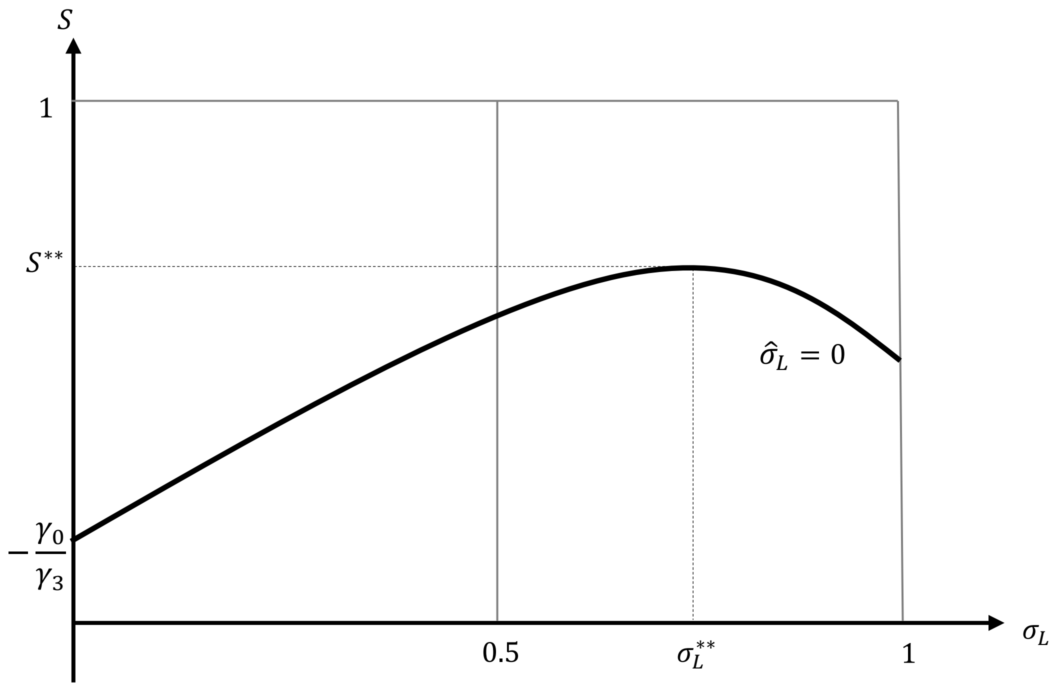
where $\gamma_0 = 1 - \sigma_{M^r} - \omega_0 + \omega_1(\beta_0 z - \rho_0 - \hat{a}_N - n) \gtreqless 0$, $\gamma_1 = \omega_1(\beta_1 z - \rho_1) - 1 < 0$, $\gamma_2 = \omega_1\alpha_2 > 0$ and $\gamma_3 = \omega_1(\beta_2 z + \alpha_3) > 0$. Here we will assume $\gamma_0 < 0$ without loss of generality. It is also assumed $\gamma_1 < 0$ which means $z < [1 + \alpha_2(1 - \sigma_{M^r})]/\beta_1\omega_1$ is the condition to be fulfilled.

Let us move on to the graphic analysis. Now we have to make some assumptions over the parameters in order to facilitate the graphic representation. First of all, we need to find the equation (33) first derivative and then its point of maximum

$$\sigma_L^{**} = -\frac{\gamma_1}{2\gamma_2} = -\frac{1}{2}\left[\frac{(\beta_1 z - \rho_1)}{\alpha_2} - \frac{1}{\omega_1\alpha_2}\right] > 0 \tag{34}$$

Again, once $\gamma_1 < 0$, we consider (34) will be positive. We will also assume $\sigma_L^{**} \in (1/2,1)$ initially without loss of generality. As the second derivative is negative, that is, $-2\gamma_2/\gamma_3 < 0$, then σ_L^{**} is a point of maximum. Therefore, given these constraints on the parameters, we make sure equation (33) falls within a meaningful economic domain, as represented by Figure 2 below. See appendix IV.

FIGURE 2 – The *locus* $\hat{\sigma}_L = 0$



The Technology Gap and Income Distribution Dynamics

First of all, we must find the non-trivial solutions for the system determined by equations (21) and (33). Taking the difference between these equations and rearranging the terms, we have

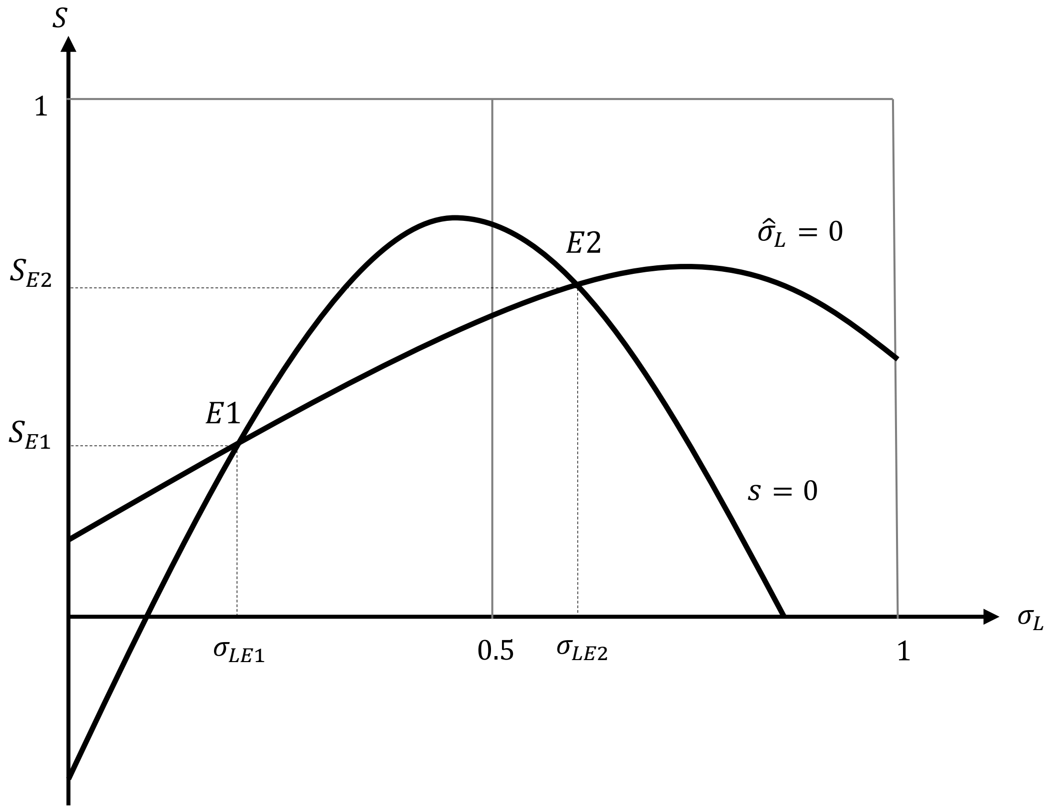
$$H(\sigma_L) = \left(\frac{\rho_0}{\alpha_3} + \frac{\gamma_0}{\gamma_3}\right) + \left(\frac{\rho_1}{\alpha_3} + \frac{\gamma_1}{\gamma_3}\right)\sigma_L - \left(\frac{\alpha_2}{\alpha_3} + \frac{\gamma_2}{\gamma_3}\right)\sigma_L^2 \tag{35}$$

The real roots of the quadratic equation (35) represent the equilibrium points where both the isoclines ($s_t = 0$ and $\hat{\sigma}_{Lt} = 0$) cancel out. Assuming $H(\sigma_L)$ has two different real roots in the meaningful domain, we obtain

$$\sigma_{LEi} = \frac{\gamma_3\rho_1 + \alpha_3\gamma_1}{2(\gamma_3\alpha_2 + \alpha_3\gamma_2)} \pm \frac{\left[(\gamma_3\rho_1 + \alpha_3\gamma_1)^2 + 4(\gamma_3\alpha_2 + \alpha_3\gamma_2)(\gamma_3\rho_0 + \alpha_3\gamma_0)\right]^{1/2}}{2(\gamma_3\alpha_2 + \alpha_3\gamma_2)} \tag{36}$$

where σ_{LEi}, for $i = \{1,2\}$, stands for generic equilibrium values of σ_L. In equilibrium $E1$, $\sigma_L = \sigma_{LE1}$ and $S(\sigma_{LE1}) = S_{E1}$ whereas in $E2$, $\sigma_L = \sigma_{LE2}$ and $S(\sigma_{LE2}) = S_{E2}$. It is assumed $\sigma_{LE1}, \sigma_{LE2}, S_{E1}, S_{E2} \in (0,1)$, so we can have meaningful results. See Figure 3 below:

FIGURE 3 – The *loci* $s = 0$ and $\hat{\sigma}_L = 0$



Now we must analyse the local stability conditions around the equilibrium points. From equations (20) and (32) we form a 2x2 non-linear dynamic system for the technology gap and income distribution. The linear version of the system is formed by the terms of the Jacobian matrix:

$$\begin{bmatrix} ds/dt \\ d\hat{\sigma}_L/dt \end{bmatrix} = \begin{bmatrix} -\alpha_3 & 2\alpha_2(\sigma_L^* - \sigma_{LEi}) \\ \theta\omega_1(\beta_2 z + \alpha_3) & 2\theta\omega_1\alpha_2(\sigma_{LEi} - \sigma_L^{**}) \end{bmatrix} \begin{bmatrix} S - S_{Ei} \\ \sigma_L - \sigma_{LEi} \end{bmatrix} \tag{37}$$

where S_{Ei}, for $i = \{1,2\}$, stands for generic equilibrium values of S. See appendix V. Moreover, from now onwards $J_{11} = -\alpha_3$, $J_{12} = 2\alpha_2(\sigma_L^* - \sigma_L)$, $J_{21} = \theta\omega_1(\beta_2 z + \alpha_3)$ and $J_{22} = 2\theta\omega_1\alpha_2(\sigma_L - \sigma_L^{**})$. First of all, an important remark must be done about J_{12}. Following Lima (2004), we can divide the domain into low wage share (LWS) and high wage share (HWS) regions. In the LWS region $\sigma_{LE} < \sigma_L^*$, which means the innovation process is wage-led, since $\partial s/\partial\sigma_L = 2\alpha_2(\sigma_L^* - \sigma_{LE}) > 0$, whereas in the HWS region $\sigma_L^* < \sigma_{LE}$, and the technological progress becomes profit-led, for $\partial s/\partial\sigma_L < 0$. As we can see in Figure 3, $E1$ is placed within the LWS region whereas $E2$ is in the HWS region.

Now we will analyse the stability conditions around the point $E1$ in the LWS region. Provided $J_{11} < 0$, the slope of the isocline $ds = 0$, given by $(-J_{12}/J_{11})$, is positive. Therefore, once $\partial s/\partial\sigma_L > 0$, an increase in σ_L increases s. It means the sign of ds is positive to the right and negative to the left of $ds = 0$. As for the isocline $d\hat{\sigma}_L = 0$, its slope given by $(-J_{22}/J_{21})$ is also positive. Once $\partial\hat{\sigma}_L/\partial s > 0$, $d\hat{\sigma}_L$ will be negative to the right and positive to the left. Thus, $E1$ in the LWS region is a saddle-point. Once the trace $Tr = J_{11} + J_{22}$ is negative, the stability condition will depend exclusively on the determinant, $Det = J_{11}J_{22} - J_{21}J_{12}$. The equilibrium point is stable if $Det > 0$. Since $E1$ is a saddle point, then the determinant must be negative. See Figure 4 below.

The same analysis can be extended to the point $E2$ in the HWS region. The slope of the isocline $ds = 0$ is now negative, for $\sigma_L^* < \sigma_{Lt}$. Since $\partial s/\partial\sigma_L < 0$, ds is negative to the right and positive to the left of the isocline $ds = 0$. Conversely, the slope of $d\hat{\sigma}_L = 0$ is still positive, for the inequality $\sigma_L^{**} > \sigma_L$ remains. Once $\partial\hat{\sigma}_L/\partial s > 0$, the sign of $d\hat{\sigma}_L$ is negative to the right and positive to the left of $d\hat{\sigma}_L = 0$. It means the point $E2$ in the HWS region is stable. We can see that in point $E2$ the trace from the Jacobian matrix is negative and the determinant is positive. See Figure 5 below.

FIGURE 4 – The low wage share region

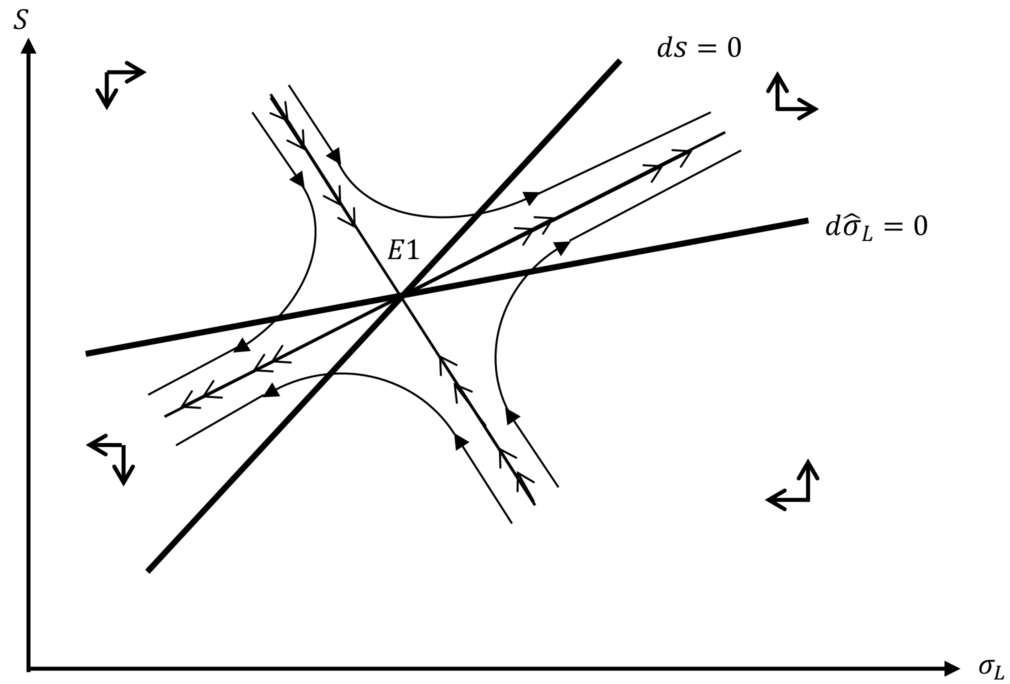
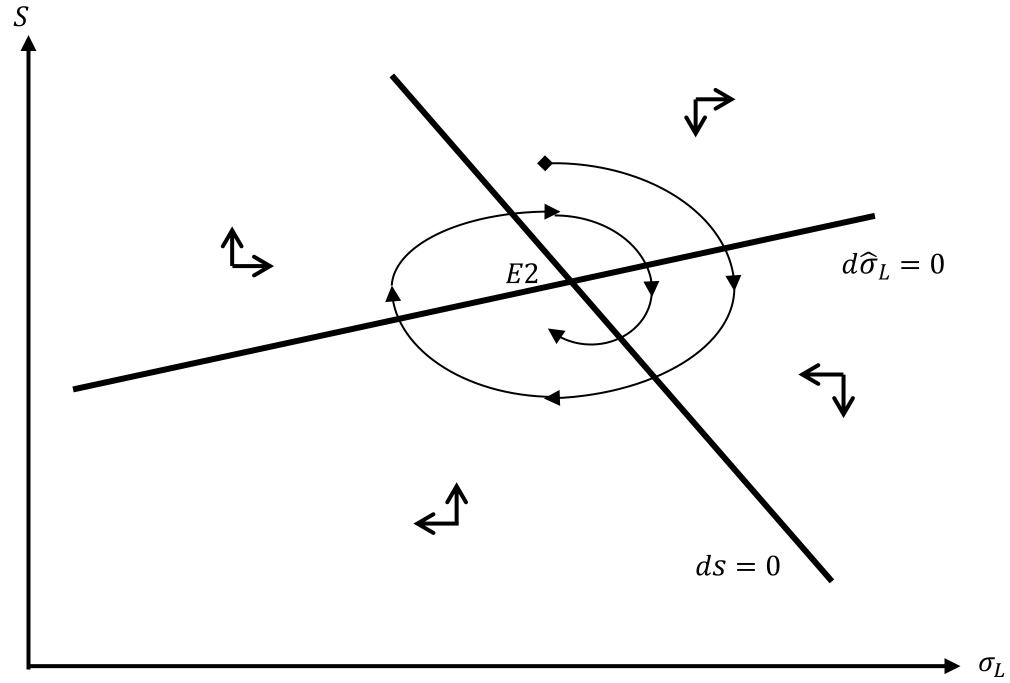


FIGURE 5 – The high wage share region



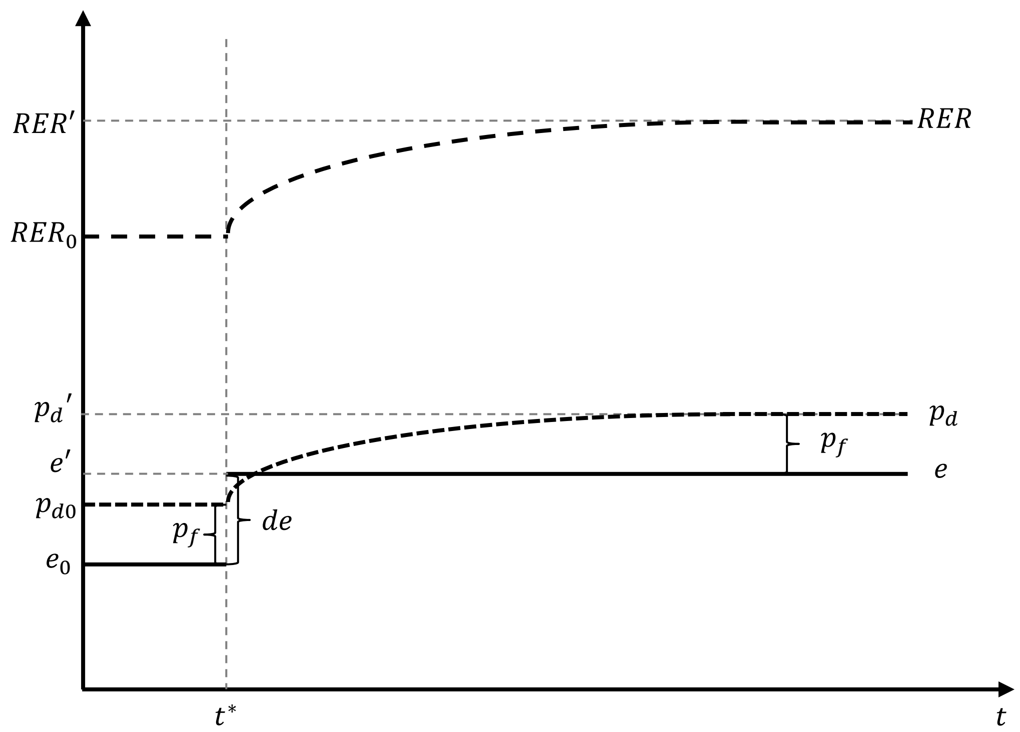
III. REAL EXCHANGE RATE, TECHNOLOGICAL PROGRESS AND INCOME DISTRIBUTION

Now let us analyse the role played by the RER on technological progress and income distribution. A money exchange rate undervaluation ($e > 0$) will improve the BPC growth rate, as long as the Marshall-Lerner condition holds, that is, $(1 + \eta + \psi) > 0$. Even though this must hold true in the short run, according to the BPC growth model, a once-for-all undervaluation is not capable of raising permanently the output growth rate unless the monetary authority continuously depreciate the home currency successively. Nonetheless, if the economy presents any positive degree of exchange rate pass-through to imported raw material prices, then changes in the money exchange rate ultimately find their way to domestic prices. It means, relative prices do not change in the long run ($e + p_f - p_d = 0$), and gains in the home country's price competitiveness are completely offset over time.

Even though the rate of change of RER diminishes through time, the same is not true if we consider the RER in level. Given a sustained rise in the rate of change of nominal exchange rate, the rate of change of domestic prices will be lagging behind at an increasing rate of change. The domestic prices adjustment process will continue until the growth rate of domestic prices matches the increased rate of change of nominal exchange rate. As a result, assuming the domestic price level does not overshoot the level of the nominal exchange rate, the RER will be higher in the long run than its initial value. Formally, variations in the level of the RER can be described as such: $dRER = RER\left(e + p_f - p_d\right)$. Initially, let us assume the Thirlwall's Law holds, and hence relative prices are constant. That is to say, we have $\left(e_0 + p_f - p_{d0}\right) = 0$, $dRER = 0$ and $RER = RER_0$; where RER_0, e_0 and p_{d0} are the initial values of the RER, nominal exchange rate growth rate and domestic prices growth rate, respectively. If it is assumed that the monetary authority – from a given point in time, say t^*, onward – undertakes a permanent increase in the nominal exchange rate growth rate, then we have

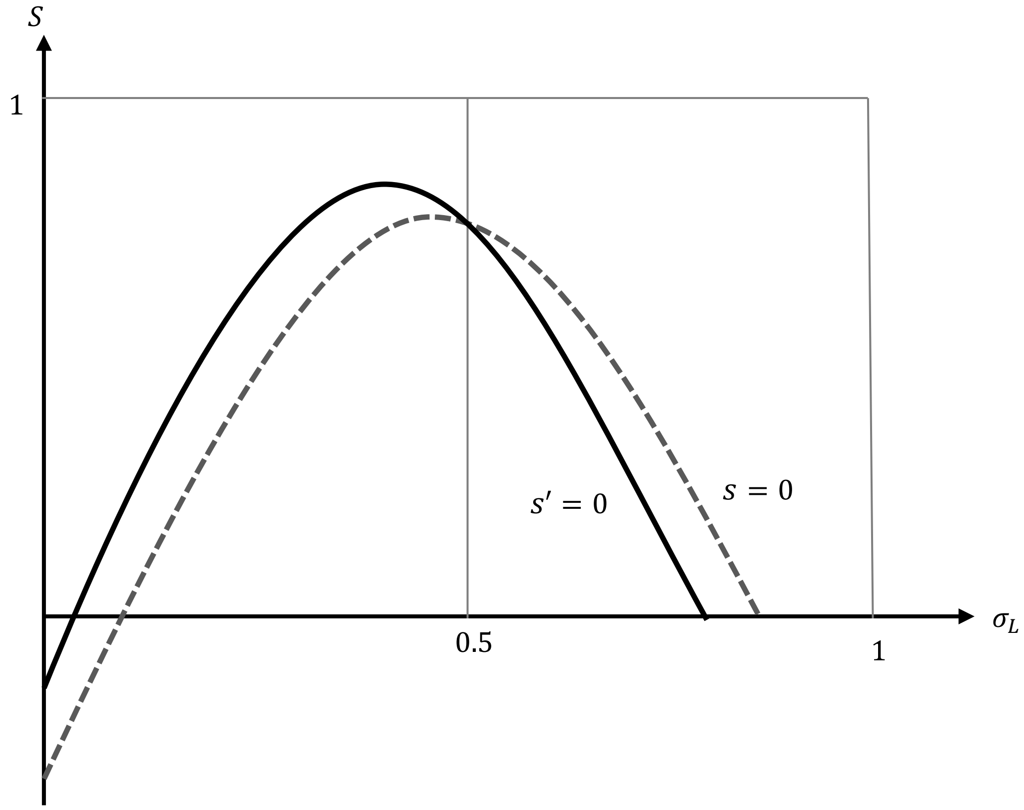
$(e + p_f - p_d) > 0$ and consequently $dRER > 0$, which means the level of the RER increases. As the real depreciation triggers the inflationary effect on domestic prices, the rate of change of the RER tends asymptotically towards zero at diminishing positive rates of change and the RER in level, consequently, increases towards a higher equilibrium level. In formal notation, we have $(e + p_f - p_d) \to 0$, $dRER \to 0$ and $RER = RER' > RER_0$, where RER' is the RER long-term level when domestic prices are fully adjusted and relative prices do not change. If another undervaluation is undertaken in a subsequent period, then we would have $RER'' > RER' > RER_0$ and so on. This is a crucial remark, for it means that every increase in the level of RER is permanent in the model. Therefore, given $\mu = Y/M^r$ remains constant over time, an undervaluation increases, *ceteris paribus*, the imported intermediate goods share on income (σ_{M^r}). Figure 6 below illustrates this mechanism laid out above and shows how permanent, exogenous shocks in the nominal exchange rate cause permanent shifts in the equilibrium RER value. See Figure 6 below:

FIGURE 6 – The permanent impact of a nominal devaluation on the RER



That said, now we must see how an undervaluation – that is, as pointed out above, an increase in the share σ_{M^r} – impacts the quadratic equations in both *loci* $s = 0$ and $\hat{\sigma}_L = 0$. As for the *locus* $s = 0$ described by equation (21), let us see first how the parabola vertex $(\sigma_L^*, S(\sigma_L^*))$ shifts after a currency depreciation. Since $d\sigma_L^*/d\sigma_{M^r} = -1/2$, a currency undervaluation reduces the point of maximum σ_L^*. On the other hand, as $dS(\sigma_L^*)/d\sigma_{M^r} = (\alpha_2/\alpha_3)[(1 - 3\sigma_M)/2] > 0$, the vertex coordinate $S(\sigma_L^*)$ will augment, once we expect $\sigma_M < 1/2$. With respect to the real roots σ_{L1} and σ_{L2}, we can say both will be reduced, as $d\sigma_{L1}/d\sigma_{M^r} \cong d\sigma_{L1}/d\sigma_{M^r} \cong d\sigma_L^*/d\sigma_{M^r} = -1/2$ and the variation of $\sqrt{\rho_1^2 + 4\rho_0\alpha_2}/2\,\alpha_2$ is negligible. Figure 7 illustrates such a dynamic.

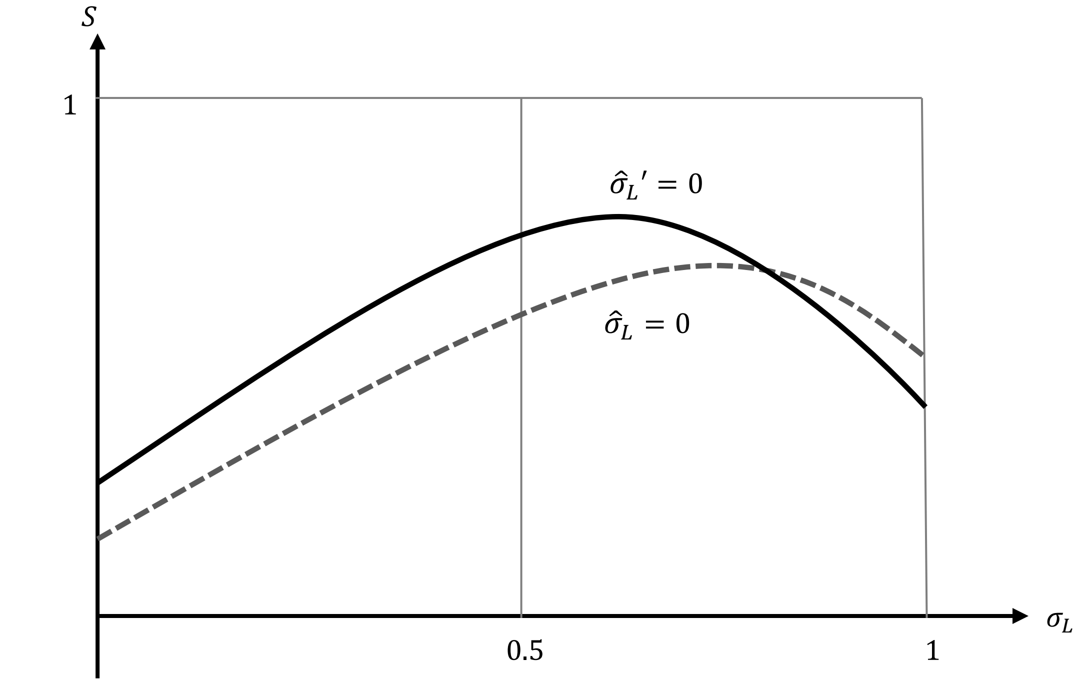
FIGURE 7 – The impact of an undervaluation in *locus* $s = 0$



The grey dotted line representing the *locus* $s = 0$ is the initial curve whereas the black line representing the *locus* $s' = 0$ is the new curve after a currency depreciation.

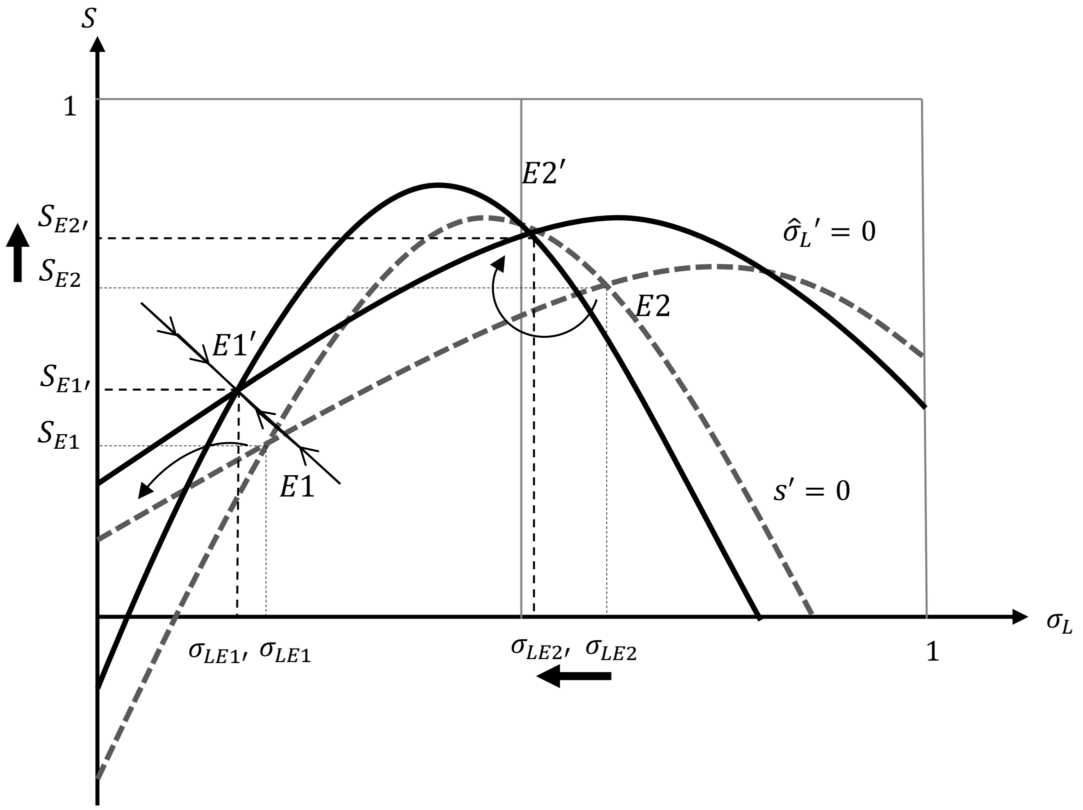
As for the *locus* $\hat{\sigma}_L = 0$ described by equation (33) we must equally start by analysing what happens with the parabola vertex $\left(\sigma_L^{**}, S(\sigma_L^{**})\right)$. Since $d\sigma_L^{**}/d\sigma_{M^r} = -1/2$ as well, an undervaluation also reduces the point of maximum σ_L^{**}. With respect to $S(\sigma_L^{**})$, we have $dS(\sigma_L^{**})/d\sigma_{M^r} = [1 + \omega_1\alpha_2(1 - 2\sigma_{M^r})/\gamma_3] - (\omega_1\,\alpha_2/2\gamma_2\gamma_3)(\gamma_1) > 0$, once $\gamma_1 < 0$ and one should expect $\sigma_{M^r} < 1/2$. As for the real roots of equation (33), since the first one is negative and therefore is outside the relevant domain, we must analyse how the intercept shifts as a result of an undervaluation. Provided the intercept $(-\gamma_0/\gamma_3)$, we have $d(-\gamma_0/\gamma_3)/d\sigma_{M^r} = [1 + \omega_1\alpha_2(1 - 2\sigma_{M^r})]/\gamma_3 > 0$, once we expect $\sigma_{M^r} < 1/2$. It means, in this example, an undervaluation will raise the intercept. Lastly, since the second real root is positive and higher than unity, we have to evaluate the behavior of the function $S(1)$ after an undervaluation. Once $dS(1)/d\sigma_{M^r} = (1 - 2\omega_1\alpha_2\sigma_M)/\gamma_3 \gtreqless 0$, then $S(1)$ can increase or decrease as a result of an undervaluation. Here we will assume $dS(1)/d\sigma_{M^r} < 0$ without loss of generality. Therefore the function $S(1)$ undergoes a downward shift caused by a currency depreciation. Figure 8 shows what happens with the locus $\hat{\sigma}_L = 0$.

FIGURE 8 – The impact of an undervaluation on *locus* $\hat{\sigma}_L = 0$



Once again, the grey dotted line $\hat{\sigma}_L = 0$ is the initial curve and the black one $\hat{\sigma}_L' = 0$ is the new locus after an undervaluation. Figure 9 bellow shows the impact of an increase in σ_{M^r} on the dynamics between technological progress and income distribution in the long run.

FIGURE 9 – The impact of an undervaluation on the between S and σ_L



Provided the point $E1$ is a saddle-point, that is, an unstable solution, if the undervaluation places the current system non-trivial solution under the separatrix, then both income distribution and technology gap will follow an ever decreasing path over time. The opposite happens if the current non-trivial solution lies over the separatrix. It means both variables under discussion will increase indefinitely.

As for the point $E2$, once it is a stable solution, then $E2$ moves to $E2'$. In such an example, we can see the undervaluation reduced the technology gap but worsened the income inequality.

IV. THE NET IMPACT OF AN UNDERVALUED REAL EXCHANGE RATE ON LONG-TERM GROWTH

In order to evaluate the net impact of an undervaluation on long-term growth we start the analysis from equation (29), as follows

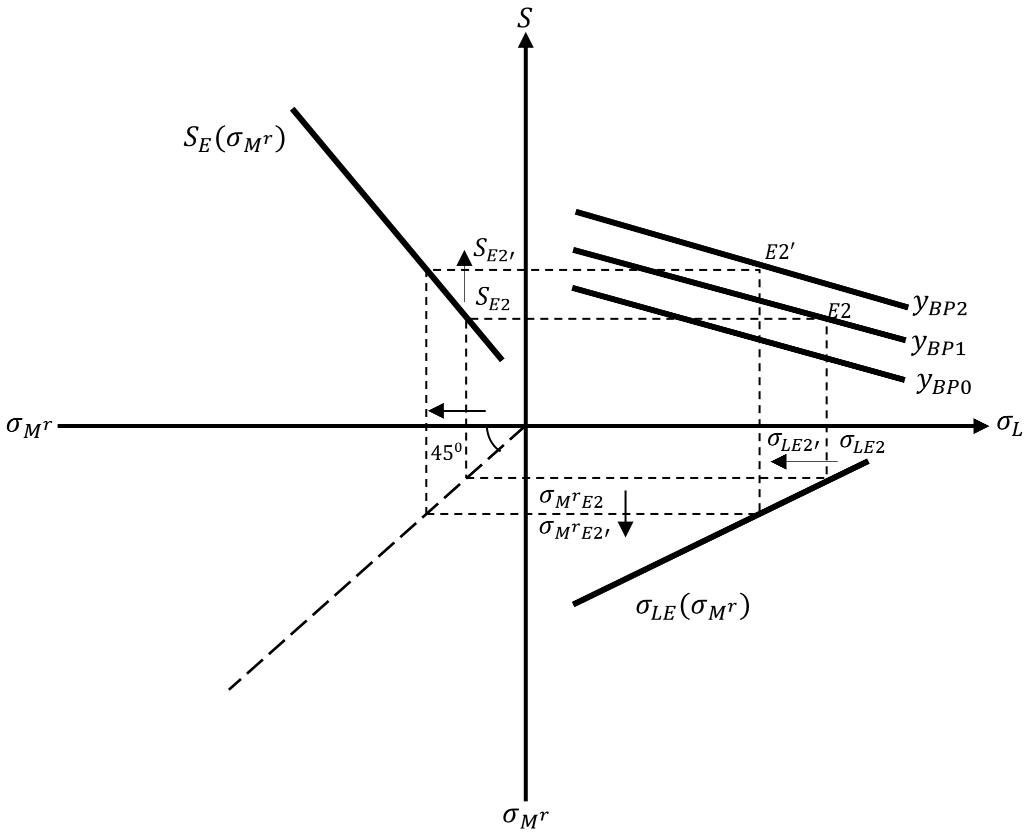
$$y_{BP} = \frac{\varepsilon}{\pi} z = (\beta_0 + \beta_1 \sigma_L + \beta_2 S) z \tag{29}$$

However, in order to evaluate if a currency depreciation will propel the long-term growth rate or not, we must consider, not only the impact of changes in the RER on the technology gap (S) and wage share (σ_L), but also the values of the parameters β_0, β_1 and β_2. First of all, since $\beta_1, \beta_2 > 0$, it is easy to see if both S and σ_L increase (decrease), then the long-term growth rate must also increase (decrease). This is the case of the equilibrium $E1$ (and $E1'$) in the LWS region, as shown in Figure 9 above. Since $E1$ is a saddle point, only displacements along the separatrix make the system to move towards the new equilibrium point $E1'$ after an undervaluation. Perturbations in any other direction will be amplified, as the system veers off the new equilibrium point. Taking the scenario represented in Figure 9 as an example, if the initial equilibrium $E1$ lies below the separatrix crossing the new equilibrium $E1'$, then an undervaluation reduces both S and σ_L, thus harming long-term growth. Conversely, if $E1$ is placed above the separatrix in $E1'$, then both S and σ_L will increase until the system eventually reaches the stable equilibrium $E2'$ in the HWS region. If that is the case, from equation (29), then it is easy to see that the long-term growth rate will continuously increase until both S and σ_L stabilize in a higher equilibrium point, say $E2'$ in this case. As we can see, if both S and σ_L are increasing (decreasing) simultaneously, then the analysis of the impact of an undervaluation on long-term growth is quite straightforward.

However, if after an undervaluation the system is either moving along the separatrix in the LWS region or shifting from $E2$ to $E2'$ in the HWS region, then we see that the equilibrium values S_{Ei} and σ_{LEi} move in opposite directions, thereby rendering an ambiguous net effect on the long-term equilibrium growth rate. For convenience, it will be considered here, from now on, only perturbations in the system taking place in the HWS region without loss of generality – since this seems to be a more general case and it is also highly unlikely the system will ever be placed precisely on the separatrix in the LWS region. However, the same analysis can be employed to account for the net impact of an undervaluation on long-term growth in the LWS region.

That said, let us now illustrate in Figure 10 below the net impact of an undervaluation on long-term growth in the HWS region.

FIGURE 10 – The impact of an undervaluation on long-term growth



In Figure 10, equation (29) is represented in the first quadrant as a set of iso-growth curves, where each of which consists of a constant equilibrium growth rate in the space (σ_L, S). The slope of these curves is given by $-\beta_1/\beta_2$.

As for the fourth quadrant, we must analyse the relationship between RER (or σ_{M^r}) and the equilibrium wage share in the HWS region (σ_{LE2}). Let us rewrite σ_{LE2} expressed in (36), as follows

$$\sigma_{LE2} = \frac{\gamma_3\rho_1 + \alpha_3\gamma_1}{2(\gamma_3\alpha_2 + \alpha_3\gamma_2)} + R \qquad (36^*)$$

where R is the rest of the right-hand side of the expression (36). If we substitute $\rho_1 = \alpha_2(1-\sigma_{M^r})$, $\gamma_1 = \omega_1(\beta_1 z - \rho_1) - 1$, $\gamma_2 = \omega_1\alpha_2$ and $\gamma_3 = \omega_1(\beta_2 z + \alpha_3)$ in (36*) and assume $dR/d\sigma_{M^r}$ is negligible for expositional ease, then the RER (σ_{M^r}) effect on σ_{LE2} is given by the differential

$$\frac{d\sigma_{LE2}}{d\sigma_{M^r}} = -\frac{\alpha_2\omega_1\beta_2 z}{2[\alpha_2\omega_1(\beta_2 z + \alpha_3) + \omega_1\alpha_2\alpha_3]} < 0 \qquad (38)$$

Therefore, from (38), it can be said that the RER (σ_{M^r}) and the equilibrium wage share in the HWS region (σ_{LE2}) are inversely related. This falls in line with the scenario represented in Figure 9, where a currency devaluation reduces the wage share ($\sigma_{LE2\prime} - \sigma_{LE2} < 0$). In Figure 10 the relationship between σ_{M^r} and σ_{LE2} is assumed to be linear for convenience. One can also think of it as a linear approximation of the actual function around the initial equilibrium point ($\sigma_{M^r}, \sigma_{LE2}$).

In order to explore the relationship between RER (σ_{M^r}) and the equilibrium technology gap level in the HWS region (S_{E2}) presented in the second quadrant of Figure 10, we must first define S as a function of the equilibrium wage share, such as $S_{E2} = S(\sigma_{LE2})$. Accordingly, by plugging the equilibrium wage share σ_{LE2} into equation (21) – it could be also in equation (33) without loss of generality, since S_{E2} is determined by the intersection between equations (21) and (33) – and rearranging the terms, we have

$$\frac{dS(\sigma_{LE2})}{d\sigma_{M^r}} = \frac{\alpha_2}{\alpha_3}\left[\underbrace{(1-\sigma_{LE2}-2\sigma_{M^r})}_{>0} + \underbrace{(1-2\sigma_{LE2}-\sigma_{M^r})}_{<0}\frac{d\sigma_{LE2}}{d\sigma_{M^r}}\right] > 0 \qquad (39)$$

We plausibly assume that the imported raw materials share is lower than the profit share in income: $1-\sigma_{LE2}-2\sigma_{M^r} = \sigma_{KE2}-\sigma_{M^r} > 0$. In the HWS region, we also have, by definition, a wage share greater than the profit share: $1-2\sigma_{LE2}-\sigma_{M^r} = \sigma_{KE2}-\sigma_{LE2} < 0$. Since, from (38), $d\sigma_{LE2}/d\sigma_{M^r}$ is negative, then the differential (39) is positive. That is to say, the higher the RER, the higher the equilibrium technology gap level. The function in the second quadrant is also assumed to be a linear approximation around the initial equilibrium point (σ_{M^r}, S_{E2}). This result agrees with the scenario illustrated in Figure 9, where a currency undervaluation increased the technology gap $(S_{E2\prime} - S_{E2} > 0)$.

Having defined the relations between the endogenous variables, we can now analyze the scenario illustrated in Figure 10. It starts in the third quadrant with an increase in the imported raw materials share in income caused by a currency devaluation $(\sigma_{M^r E2\prime} - \sigma_{M^r} > 0)$. In the fourth quadrant, we can see that the devaluation reduces the wage share in income $(\sigma_{LE2\prime} - \sigma_{LE2} < 0)$. The magnitude of such impact depends on the parameters in the differential (38). On the other hand, according to the parameters in the differential (39), the same devaluation increases the technological progress in the home country $(S_{E2\prime} - S_{E2} > 0)$, as shown in the second quadrant. In the first quadrant, since the iso-growth curves are sufficiently elastic – or the slope of the iso-growth curves is sufficiently low in absolute value $|\beta_1/\beta_2|$ – we can say that a currency devaluation spurs growth in the long run $(y_{BP2} - y_{BP1} > 0)$.

To sum up, this model shows that the net effect of a currency devaluation on long-term growth is ambiguous. It depends not only on the parameters of the model, but also on the magnitude in absolute values of changes in its endogenous variables, such as the technology gap and the wage share, and on the type of growth regime, that is wage-led or profit-led. Therefore, policymakers should take into account the idiosyncrasies concerning all these issues with respect to the country under consideration before they decide to promote economic recovery or simply boost growth by the use of currency devaluations.

V. SUMMARY

This paper contributes to the post-Keynesian literature on BPC growth, distribution and technological innovation by developing a theoretical framework in which the specialization pattern of the economy is determined by the dynamics between technological progress and income distribution. An increase in the pace of technological innovation induces to improvements in the non-price competitiveness of domestic goods. On the other hand, an increase in the wage share in total income lower capitalists' saving, and consequently reduces their capability of consuming superfluous and/or highly technological imported consumption goods.

In terms of policy, although the literature has been emphasizing the importance of a devalued RER for output growth, exchange rate impacts on the specialization pattern of the economy via simultaneous changes in income distribution and technological change have been neglected. This paper also contributes to the BPC growth literature by exploring at length the links between a currency devaluation and improvements in non-price competitiveness of domestic goods. It establishes a set of conditions under which a currency devaluation can boost or harm growth in the long run. In a LWS region, it is more likely that technical change and wage share will move in the same direction after a devaluation, thereby impacting on growth accordingly. On the other hand, in a HWS region, a devaluation will cause technological progress and wage share to move in opposite directions, which means that the net impact of a real devaluation on long-term growth is ambiguous and depends on the parameters of the model.

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Appendix I

Proposition: given $\tau = 0$, in the long term $\varphi = \bar{\varphi} \;\Rightarrow\; p_d = p_f + e$.

<u>Proof:</u>

$$\varphi = \frac{P^W}{P^E + P^W} \Rightarrow \frac{1}{\varphi} = 1 + \frac{P^E}{P^W}$$

If P^E/P^W is constant, then φ is constant as well. Therefore:

$$\frac{P^E}{P^W} = k \text{ or } ln\left(\frac{P^E}{P^W}\right) = ln(c)$$

where $c > 0$ is a constant. Taking the differential with respect to time, we have:

$$p^E = p^W$$

Until now it is proven that $\varphi = \bar{\varphi} \Rightarrow p^E = p^W$. The next step is to prove the following:

$$p^E = p^W \Rightarrow p_d = p_f + e$$

Rewriting equation (23) and considering $\tau = 0$ in the long term, we obtain:

$$p_d = \varphi p^W + (1 - \varphi)p^E \qquad (*)$$

wherein $p^W = w - r$ and $p^E = p_f + e$, according to the equations (27) and (26).
This way, considering that $\varphi = \bar{\varphi} \Rightarrow p^E = p^W$ in $(*)$, we have:
$p_d = p^W = p^E = p_f + e$, which had to be demonstrated.

APPENDIX II

Equation (19):
$s = \alpha_0 + \alpha_1\sigma_K - \alpha_2\sigma_K^2 - \alpha_3 S$

If $\sigma_L = 1 - \sigma_L - \sigma_M$, then:
$s = \alpha_0 + \alpha_1(1 - \sigma_L - \sigma_M) - \alpha_2(1 - \sigma_L - \sigma_M)^2 - \alpha_3 S$
$s = (\alpha_0 + \alpha_1 - \alpha_2) + (-\alpha_1 + 2\alpha_2)\sigma_L - \alpha_2\sigma_L^2 + (-\alpha_1 + 2\alpha_2)\sigma_M - \alpha_2\sigma_M^2 - \alpha_2\sigma_L\sigma_M - \alpha_3 S$
Assuming $\alpha_1 = \alpha_2$:
$s = [\alpha_0 + \alpha_2(\sigma_M - \sigma_M^2)] + \alpha_2\sigma_L - \alpha_2\sigma_L^2 - \alpha_2\sigma_L\sigma_M - \alpha_3 S$
where $\rho_0 = \alpha_0 + \alpha_2(\sigma_M - \sigma_M^2)$. Then:
$s = \rho_0 + \alpha_2[(1 - \sigma_M)\sigma_L - \sigma_L^2] - \alpha_3 S$
By making $\rho_1 = \alpha_2(1 - \sigma_M)$, we have equation (28):
$s = \rho_0 + \rho_1\sigma_L - \alpha_2\sigma_L^2 - \alpha_3 S$

APPENDIX III

Relation between the technological gap and income distribution within the locus $s = 0$:
$S = \frac{\rho_0}{\alpha_3} + \frac{\rho_1}{\alpha_3}\sigma_L - \frac{\alpha_2}{\alpha_3}\sigma_L^2$
This appendix is twofold. First, we will analyse the conditions with respect to the domain of σ_L. Later, we ought to do the same regarding $S(\sigma_L^*)$.

A) Domain of σ_L

Condition to obtain two distinct real roots ($\Delta > 0$):
$\rho_0 + \rho_1\sigma_L - \alpha_2\sigma_L^2 = 0$
Given $\rho_0 < 0$ and $\alpha_2 > 0$, the following condition must be fulfilled:
If $\Delta > 0$, then $\rho_1^2 > -4\rho_0\alpha_2$.
A.1) First root: $\sigma_{L1} \in (0,1)$

Condition I: $\sigma_{L1} > 0$
$\sigma_{L1} = \frac{-\rho_1 + \sqrt{\rho_1^2 + 4\rho_0\alpha_2}}{-2\alpha_2} = \sigma_L^* - \frac{\sqrt{\rho_1^2 + 4\rho_0\alpha_2}}{2\alpha_2} > 0$
$\alpha_2(1 - \sigma_M) > \sqrt{\rho_1^2 + 4\rho_0\alpha_2}$
$\rho_1^2 > \rho_1^2 + 4\rho_0\alpha_2$. TRUE, once we are assuming $\rho_0 < 0$.

Condition II: $\sigma_{L1} < 1$
TRUE, once $0 < \sigma_{L1} < \sigma_L^* < 1/2$

A.2) Second root: $\sigma_{L2} \in (0,1)$

Condition I: $\sigma_{L2} > 0$

$$\sigma_{L2} = \frac{-\rho_1 - \sqrt{\rho_1^2 + 4\rho_0\alpha_2}}{-2\alpha_2} = \sigma_L^* + \frac{\sqrt{\rho_1^2 + 4\rho_0\alpha_2}}{2\alpha_2} > 0$$

Since all the parameters are positives, the statement is TRUE. Thus $\sigma_{L2} > 0$.

Condition II: $\sigma_{L2} < 1$

$$\sigma_{L2} = \frac{-\rho_1 - \sqrt{\rho_1^2 + 4\rho_0\alpha_2}}{-2\alpha_2} < 1$$

If $\sigma_{L2} \in (\sigma_L^*, 1)$, then $\sigma_{L2} - \sigma_L^* < 1 - \sigma_L^*$. Therefore:

$$\sigma_L^* + \frac{\sqrt{\rho_1^2 + 4\rho_0\alpha_2}}{2\alpha_2} - \sigma_L^* < 1 - \frac{(1 - \sigma_M)}{2}$$

$$\sqrt{\rho_1^2 + 4\rho_0\alpha_2} < \alpha_2(\sigma_M + 1)$$

$$\alpha_2^2(1 - \sigma_M)^2 + 4\rho_0\alpha_2 < \alpha_2^2(\sigma_M + 1)^2$$

$$4\rho_0 < \alpha_2[(\sigma_M + 1)^2 - (1 - \sigma_M)^2]$$

By considering:

$$[(\sigma_M + 1)^2 - (1 - \sigma_M)^2] = [(\sigma_M + 1) + (1 - \sigma_M)][(\sigma_M + 1) - (1 - \sigma_M)]$$

and then rearranging the terms, we have:

$$\rho_0 < \alpha_2\sigma_M$$

From appendix I we know $\rho_0 = \alpha_0 + \alpha_2(\sigma_M - \sigma_M^2)$. Then:

$$\alpha_0 + \alpha_2(\sigma_M - \sigma_M^2) < \alpha_2\sigma_M$$

$$\alpha_0 - \alpha_2\sigma_M^2 < 0$$

Therefore, once $\alpha_0 < 0$, then this condition is TRUE. In this work we will assume such a condition for the sake of the graphic exposition.

B) Domain of $S(\sigma_L^*)$

$$S(\sigma_L^*) = \frac{\rho_0}{\alpha_3} + \frac{\rho_1}{\alpha_3}\frac{(1 - \sigma_M)}{2} - \frac{\alpha_2}{\alpha_3}\frac{(1 - \sigma_M)^2}{4}$$

Substituting $\rho_1 = \alpha_2(1 - \sigma_M)$ in the equation above and rearranging the terms, we obtain:

$$S(\sigma_L^*) = \frac{\rho_0}{\alpha_3} + \frac{\alpha_2}{\alpha_3}(\sigma_L^*)^2$$

B.1) $S(\sigma_L^*) > 0$

The condition is

$$\frac{\rho_0}{\alpha_3} + \frac{\alpha_2}{\alpha_3}(\sigma_L^*)^2 > 0 \Rightarrow \rho_0 > -\alpha_2(\sigma_L^*)^2$$

As $\rho_0 = \alpha_0 + \alpha_2(\sigma_M - \sigma_M^2)$, we have:

$$\alpha_0 + \alpha_2(\sigma_M - \sigma_M^2) < \alpha_3 - \alpha_2\frac{(1 - \sigma_M)^2}{4}$$

Rearranging the terms:

$$\alpha_2\sigma_M(2 - 3\sigma_M) > (-4\alpha_0 - \alpha_2)$$

If this inequality holds, then $S(\sigma_L^*) > 0$.

B.2) $S(\sigma_L^*) < 1$

The condition is

$$\frac{\rho_0}{\alpha_3} + \frac{\alpha_2}{\alpha_3}(\sigma_L^*)^2 < 1 \Rightarrow \rho_0 < \alpha_3 - \alpha_2(\sigma_L^*)^2$$

As $\rho_0 = \alpha_0 + \alpha_2(\sigma_M - \sigma_M^2)$, we have:

$$\alpha_0 + \alpha_2(\sigma_M - \sigma_M^2) < \alpha_3 - \alpha_2\frac{(1-\sigma_M)^2}{4}$$

Rearranging the terms:

$$\alpha_2\sigma_M(2 - 3\sigma_M) < (4\alpha_3 - 4\alpha_0 - \alpha_2)$$

We expect the term $\alpha_2\sigma_M(2 - 3\sigma_M)$ must be positive but quite small, once we also expect $\sigma_M < 2/3$. It means the condition under discussion only holds if the term $(4\alpha_3 - 4\alpha_0 - \alpha_2)$ is sufficiently positively high. If we assume $\alpha_0 < 0$, then the condition can be reasonably fulfilled. Therefore, $S(\sigma_L^*) < 1$.

APPENDIX IV

Relation between the technological gap and income distribution within the locus $\hat{\sigma}_{Lt} = 0$:

$$S = -\frac{\gamma_0}{\gamma_3} - \frac{\gamma_1}{\gamma_3}\sigma_L - \frac{\gamma_2}{\gamma_3}\sigma_L^2$$

Following the appendix II, this appendix is also twofold. First, we will analyze the conditions with respect to the domain of σ_L. Later, we will do the same regarding $S(\sigma_L^{**})$.

A) Domain of σ_L

Condition to obtain two distinct real roots ($\Delta > 0$):

$$-\gamma_0 - \gamma_1\sigma_L - \gamma_2\sigma_L^2 = 0$$

Given $\gamma_0, \gamma_1 < 0$ and $\gamma_2 > 0$, the following condition must be fulfilled:

If $\Delta > 0$, then $(-\gamma_1)^2 > 4\gamma_0\gamma_2$.

A.1) First root: $\sigma_{L1} \in (0,1)$

<u>Condition I</u>: $\sigma_{L1} > 0$

$$\sigma_{L1} = \frac{\gamma_1 + \sqrt{(-\gamma_1)^2 - 4\gamma_0\gamma_2}}{-2\gamma_2} = \sigma_L^{**} - \frac{\sqrt{(-\gamma_1)^2 - 4\gamma_0\gamma_2}}{2\gamma_2} > 0$$

$(-\gamma_1)^2 < (-\gamma_1)^2 - 4\gamma_0\gamma_2$. FALSE, once we are assuming $\gamma_0 < 0$. Once the intercept $-\gamma_0/\gamma_3 > 0$ it can be easily seen the root σ_{L1} must be negative.

<u>Condition II</u>: $\sigma_{L1} < 1$

FALSE. Since condition I above does not hold, condition II is false by definition.

A.2) Second root: $\sigma_{L2} \in (0,1)$

<u>Condition I</u>: $\sigma_{L2} > 0$

$$\sigma_{L2} = \frac{\gamma_1 - \sqrt{(-\gamma_1)^2 - 4\gamma_0\gamma_2}}{-2\gamma_2} = \sigma_L^{**} + \frac{\sqrt{(-\gamma_1)^2 - 4\gamma_0\gamma_2}}{2\gamma_2} > 0$$

Since all the parameters are positives, the statement is TRUE. Thus $\sigma_{L2} > 0$.

<u>Condition II</u>: $\sigma_{L2} < 1$

We know $\sigma_L^{**} < \sigma_{L2} < 1$, once, by assumption, $\sigma_L^{**} < 1$. Therefore:

$$\sigma_{L2} - \sigma_L^{**} < 1 - \sigma_L^{**}$$
$$\frac{\sqrt{(-\gamma_1)^2 - 4\gamma_0\gamma_2}}{2\gamma_2} < 1 + \frac{\gamma_1}{2\gamma_2}$$
$$\sqrt{(-\gamma_1)^2 - 4\gamma_0\gamma_2} < 2\gamma_2 + \gamma_1$$
$$(-\gamma_1)^2 - 4\gamma_0\gamma_2 < (2\gamma_2 + \gamma_1)^2$$
$$\gamma_2(1 + \gamma_1 + \gamma_0) > 0$$
Once $\gamma_2 > 0$, we must analyze if $1 + \gamma_1 + \gamma_0 > 0$. Dividing this term by $-2\gamma_2$, we have:
$$\sigma_L^{**} < \frac{1 + \gamma_0}{2\gamma_2}$$
In this work we will assume this condition is not fulfilled without loss of generality. Therefore $\sigma_{L2} > 1$.

B) Domain of $S(\sigma_L^{**})$

Allowing σ_L^{**} into equation (37) and rearranging the terms:
$$S(\sigma_L^{**}) = \gamma_0 + \gamma_1\left(\frac{\gamma_1}{2\gamma_2}\right) + \gamma_2\left(\frac{\gamma_1}{2\gamma_2}\right)^2 = \gamma_0 + \frac{\gamma_1^2}{2}$$

B.1) $S(\sigma_L^{**}) > 0$

$$\gamma_0 + \frac{\gamma_1^2}{2} > 0 \Rightarrow \gamma_0 > -\frac{\gamma_1^2}{2}$$

B.2) $S(\sigma_L^{**}) < 1$

$$\gamma_0 + \frac{\gamma_1^2}{2} < 1 \Rightarrow \gamma_0 < 1 - \frac{\gamma_1^2}{2}$$

APPENDIX V

$$J_{11} = -\alpha_3$$
$$J_{12} = \rho_1 - 2\alpha_2 = \alpha_2(1 - \sigma_M - 2\sigma_L) = 2\alpha_2(\sigma_L^* - \sigma_L)$$
$$J_{21} = \theta\gamma_3 = \theta\omega_1(\beta_2 z + \alpha_3)$$
$$J_{22} = \theta(2\gamma_2\sigma_L + \gamma_1) = 2\theta\omega_1\alpha_2\sigma_L + \theta[\omega_1(\beta_1 z - \rho_1) - 1] = 2\theta\omega_1\alpha_2(\sigma_L - \sigma_L^{**})$$